Determining the Amplitude of Mass Fluctuations in the Universe

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Abstract

We present a method for determining the *rms* mass fluctuations on 8 h^{-1}Mpc scale, σ_8. The method utilizes the rate of evolution of the abundance of rich clusters of galaxies. Using the Press-Schechter approximation, we show that the cluster abundance evolution is a strong function of σ_8: $d\log n/dz \propto -1/\sigma_8^2$; low σ_8 models evolve exponentially faster than high σ_8 models, for a given mass cluster. For example, the number density of Coma-like clusters decreases by a factor of $\sim 10^3$ from $z=0$ to $z \simeq 0.5$ for σ_8=0.5 models, while the decrease is only a factor of ~ 5 for $\sigma_8 \simeq 1$. The strong exponential dependence on σ_8 arises because clusters represent rarer density peaks in low σ_8 models. We show that the evolution rate at $z \lesssim 1$ is insensitive to the density parameter Ω or to the exact shape of the power spectrum. Cluster evolution therefore provides a powerful constraint on σ_8. Using available cluster data to $z \sim 0.8$, we find $\sigma_8 = 0.83 \pm 0.15$. This amplitude implies a bias parameter $b \simeq \sigma_8^{-1} = 1.2 \pm 0.2$, i.e., a nearly unbiased universe with mass approximately tracing light on large scales.

subject headings : galaxies : clusters – galaxies : evolution – galaxies : formation – cosmology : theory – large scale structure of universe

1. Introduction

Density fluctuations in the early universe provide the initial seeds for the structures we see today. Various observations of large-scale structure constrain the *shape* of the power spectrum of fluctuations on different scales (Fisher *et al.* 1993, Park *et al.* 1994, Peacock & Dodds 1994, Strauss & Willick 1996, Landy *et al.* 1996), but the *amplitude* of the mass fluctuation spectrum has been difficult to obtain. (The microwave background anisotropy provides so far the only direct amplitude measure on large scales; Smoot *et al.* 1992, Bunn & White 1997). Knowledge of both the shape and amplitude of the spectrum is needed before critical cosmological implications can be derived.

The amplitude of the mass fluctuations provides an important constraint on the relative distribution of mass and light in the universe, which remains one of the central unresolved problems in cosmology: does mass follow light on large scales (an unbiased universe) or is mass distributed differently than light (biased universe)? The bias parameter (Kaiser 1984) is quantified as $b = \delta_{gal}/\delta_m \simeq \sigma_8(gal)/\sigma_8 \simeq 1/\sigma_8$, where $\delta = \Delta\rho/\overline{\rho}$ is the overdensity (in galaxies or mass, respectively), σ_8 is the *rms* mass fluctuation within a top-hat radius of $8\mathrm{h}^{-1}$Mpc, and $\sigma_8(gal) \simeq 1$ is the observed optical galaxy fluctuations on that scale (Davis & Peebles 1983). A bias of $b \simeq \sigma_8 \simeq 1$ implies an unbiased universe, where mass follows light on these scales (assuming a scale-independent bias on large scales), while $b \simeq 2$ ($\sigma_8 \simeq 0.5$) implies a highly biased universe, with mass distributed more widely than light.

Galaxy clusters are useful tools in cosmology. Since clusters represent high density peaks, their present-day abundance directly reflects the amplitude of density fluctuations on the relevant cluster mass scale; the latter corresponds to the mass enclosed within $R \sim 8\mathrm{h}^{-1}$Mpc spheres in the early universe. Observations of the present-day cluster abundance indeed place one of the strongest constraints on cosmology and the amplitude of mass fluctuations : $\sigma_8\Omega^{0.5} \simeq 0.5$ (Bahcall & Cen 1992, White *et al.* 1993, Viana & Liddle 1996, Eke *et al.* 1996, Pen 1997). This relation, while powerful, is degenerate in Ω and σ_8; neither parameter can be directly determined from this constraint. Models with $\Omega = 1$ and $\sigma_8 \simeq 0.5$ are indistinguishable from models with $\Omega \simeq 0.25$ and $\sigma_8 \simeq 1$.

In the present paper we investigate a method that can measure, for the first time, the amplitude σ_8, nearly independent of Ω. The method is based on the evolution rate of cluster abundance to $z \sim 1$; this evolution breaks the degeneracy that generally exists between Ω and σ_8 and allows the first determination of each of the parameters independently (see also Press & Schechter 1974, Peebles *et al.* 1989, Henry *et al.* 1992, Eke *et al.* 1996, Oukbir & Blanchard 1997, Carlberg *et al.* 1997, Bahcall, Fan & Cen, 1997). In our previous paper (Bahcall *et al.* 1997) we presented results from cosmological simulations. In the current paper, we develop the theoretical basis that shows that the evolution rate of cluster abundance is strongly dependent on σ_8 and insensitive to Ω or to the exact shape of the power spectrum (to $z \lesssim 1$). Therefore σ_8 can be determined by observations of cluster evolution. We present the method in §2, and determine σ_8 in §3.

2. Dependence of Cluster Evolution on Cosmological Parameters

The Press-Schechter (1974) formalism approximates the evolution of the comoving number density of halos with mass M (*mass function*) in an initial Gaussian density field assuming hierarchical structure formation from linear perturbations:

$$\frac{dn}{dM} = \left(\frac{2}{\pi}\right)^{\frac{1}{2}} \frac{\overline{\rho}}{M^2} \frac{\delta_c}{\sigma(z, M)} \left| \frac{d \ln \sigma(z, M)}{d \ln M} \right| \exp\left(\frac{-\delta_c^2}{2\sigma(z, M)^2} \right), \tag{1}$$

where $\sigma(z, M)$ is the linear theory *rms* mass density fluctuation in spheres of mass M at redshift z, $\delta_c \simeq 1.68$ is the critical density contrast needed for collapse (only weakly dependent on Ω, Eke *et al.* 1996, Kitayama & Suto 1996), and $\overline{\rho}$ is the mean cosmic density. The mass refers to the virial mass of the system. Here

$$\sigma(z, M) = \frac{\sigma_0(M) D(z, \Omega, \Lambda)}{D(z = 0, \Omega, \Lambda)}, \tag{2}$$

where D is the linear growth factor (Peebles 1980) that represents the growth of a perturbation in the linear regime. For $\Omega = 1$, $D \propto (1+z)^{-1}$; D grows slower for $\Omega < 1$. $\sigma_0(M)$ is the present *rms* mass fluctuation within sphere of mass M,

$$\sigma_0^2(M) = \frac{1}{2\pi^2} \int dk k^2 P(k) |W_R(k)|^2, \tag{3}$$

where $P(k)$ is the mass density power spectrum and $W_R(k)$ is the window function. If $P(k)$ is approximated as a power law $P(k) \propto k^n$ around 8 h^{-1} Mpc scale then

$$\sigma_0(M) = \sigma_8 \left(\frac{M}{M_8}\right)^{-\alpha} \propto \quad \sigma_8 M^{-\alpha} \Omega^{\alpha}, \tag{4}$$

where $\alpha = (n+3)/6$, M_8 is the mean mass within a sphere of radius 8h^{-1}Mpc, and $M_8 \propto \Omega$. For a CDM-type spectrum, n ≈ -1 to -2 on scales of $\sim$ 8h^{-1} Mpc; for mixed (cold + hot) dark matter models, MDM, n ~ -2.5 (Klypin *et al.* 1993).

We calculate the cluster evolution by numerically integrating eq.(1) (see below). First, however, we show how eq.(1) can be approximated, in order to better understand the major factors that dominate the evolution. For clusters of Richness $\gtrsim 1$ ($M > 3 \times 10^{14} \mathrm{h}^{-1} \mathrm{M}_{\odot}$), the exponential part of eq.(1) dominates the mass function. Therefore,

$$\frac{dn}{dM} \propto \ exp\left(\frac{-\delta_c^2}{2\sigma(z, M)^2} \right) \tag{5}$$

Transferring to an integrated mass function, we find approximately (c.f., Pen, 1997)

$$\begin{aligned} \ln n(z, > M) &\propto -(D(z, \Omega, \Lambda)\sigma_0(M))^{-2} \\ &\propto -\sigma_8^{-2} \Omega^{-2\alpha} D^{-2} M^{2\alpha} \end{aligned} \tag{6}$$

We define the evolution rate $\mathcal{E}(z)$, for clusters with mass $> M$, as :

$$\mathcal{E}(z, > M) = -\frac{d \log_{10} n(z, > M)}{dz} \tag{7}$$

For small redshifts, the cluster abundance decreases approximately as $n(z) \propto 10^{-\mathcal{E}(z)(1+z)}$, with $n(z)/n(0) = 10^{-\mathcal{E}(z)z}$ (see below). For masses of rich clusters, eq.(6) yields:

$$\mathcal{E}(z, > M) \propto \sigma_8^{-2} \Omega^{-2\alpha} \frac{dD^{-2}(z, \Omega, \Lambda)}{dz} M^{2\alpha} \tag{8}$$

The evolution of the cluster abundance is therefore controlled by four factors :

1. σ_8; the evolution rate depends exponentially on σ_8^{-2}; clusters in low σ_8 models evolve exponentially faster than in high σ_8 models (for a given Ω). This arises since a fixed mass cluster corresponds to a higher relative density peak (i.e., rarer fluctuation) in low σ_8 models. This is the dominant factor in the cluster evolution at recent times.

2. dD^{-2}/dz; a linear perturbation grows slower in a low-density universe than in a high-density one. For $\Omega = 1$, the fluctuations grow as $(1+z)^{-1}$, thus continuing to evolve strongly at low z, while a low-density universe freezes out fluctuations at an earlier epoch. High Ω models thus evolve faster than lower Ω models in recent times. For the same Ω, dD^{-2}/dz is larger in a Λ-dominated universe than in an open universe (Peebles 1980), therefore stronger evolution is expected in a Λ-dominated model (see also §3).

3. $\Omega^{-2\alpha}$; this factor arises because a fixed cluster mass M, corresponds to a different part of the power spectrum $P(k)$, since $k^{-1} \sim R \sim (M/\Omega)^{1/3}$ is larger for low Ω models. According to eq.(4), $\sigma_0(M) \propto \Omega^{\alpha}$. For a given σ_8, a fixed mass cluster corresponds to a smaller $\sigma_0(M)$ for a lower Ω, implying a rarer peak in the density field. Contrary to factor #2 above, the contribution from this term results in faster evolution for low Ω. As we show below , the contributions from these two terms nearly cancel each other for $z \lesssim 1$. This cancellation can be also estimated directly from the approximate relations at low z, $dD^{-2}/dz \propto f(\Omega) \simeq \Omega^{4/7} \simeq \Omega^{0.57}$ (Lightman & Schechter 1990), and $\Omega^{-2\alpha} \propto \Omega^{-0.54}$ (for $\alpha = 0.27$, or $n = -1.4$), which approximately cancel each other.

4. $M^{2\alpha}$; this term is a constant for a given cluster mass. The evolution is stronger for more massive clusters.

We illustrate the above results in Figure 1, where the dependence of the evolution rate $\mathcal{E}(z)$ on Ω and σ_8 is calculated by numerically integrating eq.(1). The results are presented for a $\Lambda = 0$ CDM model at redshifts 0 and 1 for clusters with virial mass $M \geq 3 \times 10^{14} \mathrm{h}^{-1} \mathrm{M}_{\odot}$ (corresponding to Abell richness class ≥ 1, Bahcall & Cen 1993). A spectrum with n=–1.4 is used, i.e. $\alpha = 0.27$ (corresponding to a power spectrum shape parameter $\Gamma = \Omega h \simeq 0.25$, Eke *et al.* 1996, Viana & Liddle 1996). At $z \sim 0$, the cluster evolution rate is mainly determined by the amplitude σ_8 and is nearly independent of Ω (or Γ, see below). For $\sigma_8 = 1$, $\mathcal{E}(z) \sim 1$, implying that the abundance of richness ≥ 1 clusters decreases by a factor of ~ 3 from $z = 0$ to $z = 0.5$ ($\sim 10^{0.5\mathcal{E}(z)}$); for $\sigma_8 = 0.5$, $\mathcal{E}(z) \sim 3.5$, and the number density decreases by a

factor of $\sim 10^2$ for the same redshift interval. $\mathcal{E}(z)$ changes by only ~ 10 % when Ω changes from 0.2 to 1; the contributions from the dD^{-2}/dz and $\Omega^{-2\alpha}$ terms described above (eq.8) approximately cancel each other. The cluster evolution rate at $z < 1$, $\mathcal{E}(z) \propto \sigma_8^{-2}$ for fixed mass clusters, can therefore be used to determine σ_8. At $z \simeq 1$, the evolution rate depends on both Ω and σ_8, but σ_8 is still the dominant factor.

Figure 2a shows the dependence of the evolution rate on the slope of the spectrum n. For a CDM-type power spectrum, n ~ -1 to –2, the evolution is nearly independent of the shape of the spectrum (for any Ω; see also White *et al.* 1993, Eke *et al.* 1996, Pen 1997). The evolution rate $\mathcal{E}(z)$ changes by $\lesssim 20\%$ when n increases from –1 to –2. An MDM model (n ~ -2.5) evolves only slightly faster than Standard CDM models at this mass. Figure 2b shows the dependence of the evolution rate on the cluster mass threshold. From eq.(8) we see that $\mathcal{E}(z) \propto \sigma_8^{-2} M^{2\alpha}$. The strong dependence of the cluster evolution on σ_8 and the milder dependence on mass threshold (for $\alpha \sim 0.3$) indicates that determining σ_8 from cluster evolution is not sensitive to reasonable uncertainties in the cluster mass; changing the cluster mass by a factor of two causes a $\lesssim 20\%$ change in σ_8 (see Fig.3 below).

Figure 3 presents the cluster abundance ratio $n(z=0)/n(z=0.5)$ as a function of σ_8 for clusters with virial mass $M \geq 3 \times 10^{14}\mathrm{h}^{-1}\mathrm{M}_\odot$ (Richness $\gtrsim 1$), and $M \geq 6 \times 10^{14}\mathrm{h}^{-1}\mathrm{M}_\odot$ (Richness $\gtrsim 2$), using an n=–1.4 power spectrum. The dots represent all values of Ω from 0.2 to 1 for each σ_8. The figure illustrates the strong dependence of the evolution rate on σ_8, and the weak dependence on Ω. The relation can be approximated *for all Ω values*, as:

$$\frac{n(0)}{n(0.5)} = \begin{cases} 10^{0.55/\sigma_8^2} \;\; (\Lambda \neq 0), & 10^{0.55\Omega^{0.15}/\sigma_8^2} \;\; (\Lambda = 0), \quad M \geq 3 \times 10^{14} \\ 10^{0.8/\sigma_8^2} \;\; (\Lambda \neq 0), & 10^{0.8\Omega^{0.15}/\sigma_8^2} \;\; (\Lambda = 0), \quad M \geq 6 \times 10^{14} \end{cases} \tag{9}$$

These relations are represented by the solid lines in Fig.3. More generally, the evolution of any rich cluster can be approximated as

$$\log_{10} n(0)/n(0.5) = 0.8 M_6^{2\alpha}/\sigma_8^2 \;\; (\Lambda \neq 0), \quad 0.8\Omega^{0.15} M_6^{2\alpha}/\sigma_8^2 \;\; (\Lambda = 0) \tag{10}$$

where M_6 is the virial mass threshold in units of $6 \times 10^{14}\mathrm{h}^{-1}\mathrm{M}_\odot$, and $\alpha \simeq 0.27$ for CDM. Observations of cluster abundance from $z = 0$ to $z \sim 0.5$ can thus determine σ_8 by either directly integrating eq.(1) (Fig.3–4) or by the approximate empirical relations (9–10).

3. Determination of σ_8

In a recent paper (Bahcall *et al.* 1997), we studied the evolution of the cluster mass function using large scale N-body simulations. Several cosmological models were studied : two Standard Cold Dark Matter models (SCDM, $\Omega = 1$, $\sigma_8 = 1.05$ and 0.53); a low-density

open CDM (OCDM, Ω=0.35, $\sigma_8 = 0.79$); a low-density Λ-dominated CDM(LCDM, Ω=0.40, $\sigma_8 = 0.80$), and two Mixed Dark Matter models (MDM, $\Omega = 1$, $\sigma_8 = 0.60$ and $\sigma_8 = 0.67$). In Figure 4 we plot the relation between the cluster abundance ratio $n(z = 0)/n(z = 0.5)$ and σ_8^{-2} as obtained from the N-body simulations, for clusters with mass $M_{1.5} \geq 6.3 \times 10^{14}\mathrm{h}^{-1}\mathrm{M}_\odot$ (within a physical radius of $1.5\mathrm{h}^{-1}$Mpc, as used in observations). and compare it with the expected evolution (§2) of $\log n(z = 0)/n(z = 0.5) \propto \sigma_8^{-2}$. The solid line is calculated from the Press-Schechter relation (eq.1.) for $\Lambda = 0$ CDM model(for the mean of all Ω's, from 0.2 to 1), for the same mass clusters (within physical radius of $1.5\mathrm{h}^{-1}$Mpc, for proper comparison with the simulations and observations; the comoving radius used in the previous paper is replaced here with the physical radius to better reflect the final observations). For the Press-Schechter approximation, the mass is scaled from virial to $1.5\mathrm{h}^{-1}$Mpc radius as follows: the overdensity within the virial radius is numerically calculated for each model (e.g., Eke *et al.* 1996, Oukbir & Blanchard 1997), thus yielding the virial radius for a given virial mass. The virial mass in eq.(1) (i.e., mass within the virial radius) is then scaled to the $1.5\mathrm{h}^{-1}$Mpc mass using the observed mass profile of $M \propto r^{0.64}$ (White *et al.* 1993, Carlberg *et al.* 1997). The results are insensitive to this transformation since the two masses (radii) are comparable. The dashed line in Fig.4 represents the best-fit to the model simulations; it agrees well with the Press-Schechter relation. Both Press-Schechter and the simulation show that the evolution rate is mostly dependent on σ_8^{-2}, and is insensitive to Ω and to the shape of the power spectrum. Figure 4 also shows that, as expected (§2), the LCDM model evolves somewhat faster than OCDM, for have similar Ω and σ_8.

The strong dependence of the cluster evolution rate on σ_8 provides a powerful method for constraining σ_8 by comparison with the observed evolution of high-mass clusters to $z \sim 0.5-1$. We use the CNOC/EMSS sample of high-redshift clusters (Carlberg *et al.* 1997, Luppino & Gioia 1995) to determine the observed evolution (see, e.g., Bahcall *et al.* 1997). We find only mild evolution of cluster abundance. For massive clusters with $M_{1.5} \geq 6.3 \times 10^{14}\mathrm{h}^{-1}\mathrm{M}_\odot$ (within physical radius of $1.5\mathrm{h}^{-1}$Mpc) we find a mean best-fit observed $\log n(z) \propto z$ relation of $\log n(z = 0)/n(z = 0.5) = 0.70 \pm 0.3(1\sigma)$. This observed evolution is compared with the $\mathcal{E}(z) - \sigma_8$ prediction in Fig.4 (shaded band). The comparison yields a powerful constraint on σ_8: $\sigma_8 = 0.83 \pm 0.15$ ($1\ \sigma$) (see also Bahcall *et al.* 1997). These results are consistent with those obtained by Carlberg *et al.* (1997). Since the cluster evolution rate is not sensitive to the shape of the power spectrum, the above determination applies to both CDM and MDM models. Low σ_8 (highly biased) models predict an extremely low abundance of high mass clusters at $z \sim 0.5$, inconsistent with the observed cluster abundance at $z \sim 0.5$.

The evolution of cluster abundance provides the first method that can directly determine the amplitude of the mass fluctuations σ_8 independently of Ω or the power spectrum. Equivalently, it provides the bias parameter on these scales. The data suggests that we

live in a nearly unbiased universe, with b $\simeq \sigma_8^{-1} \simeq 1.2 \pm 0.2$, and that mass approximately traces light on large scales. The determination of σ_8 breaks the degeneracy that exists between σ_8 and Ω from the present-day cluster abundance(§1 references; Bahcall *et al.* 1997). Using the $\sigma_8 - \Omega$ relation determined by Eke *et al.* (1996), and the present constraint of $\sigma_8 = 0.83 \pm 0.15$, we find $\Omega = 0.3 \pm 0.1$.

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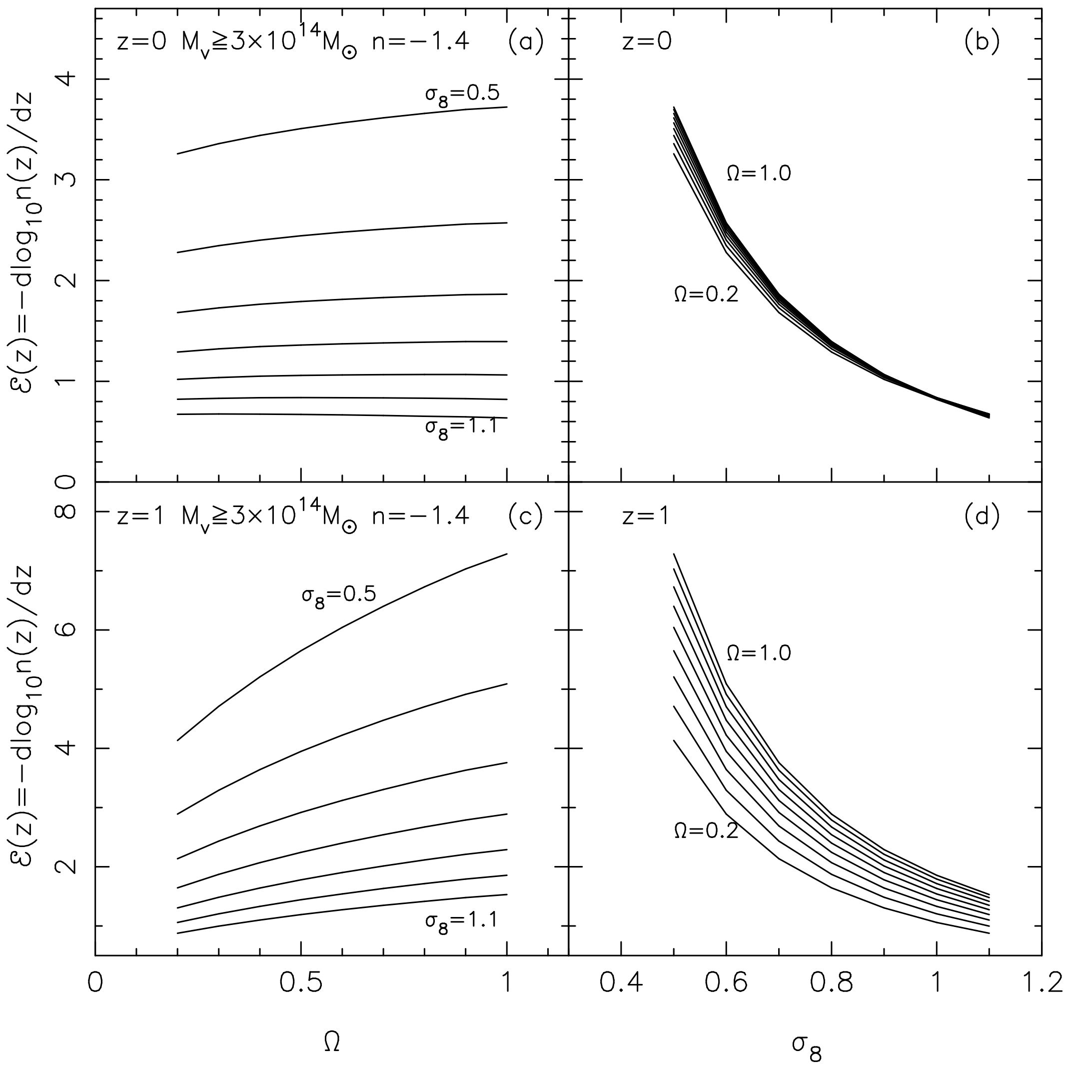

Figure 1. Dependence of the evolution rate $\mathcal{E}(z)$ of $M_v \geq 3 \times 10^{14}\mathrm{h}^{-1}\mathrm{M}_{\odot}$ clusters on Ω and σ_8 for $\Lambda = 0$ models (at $z \sim 0$ and $z \sim 1$).

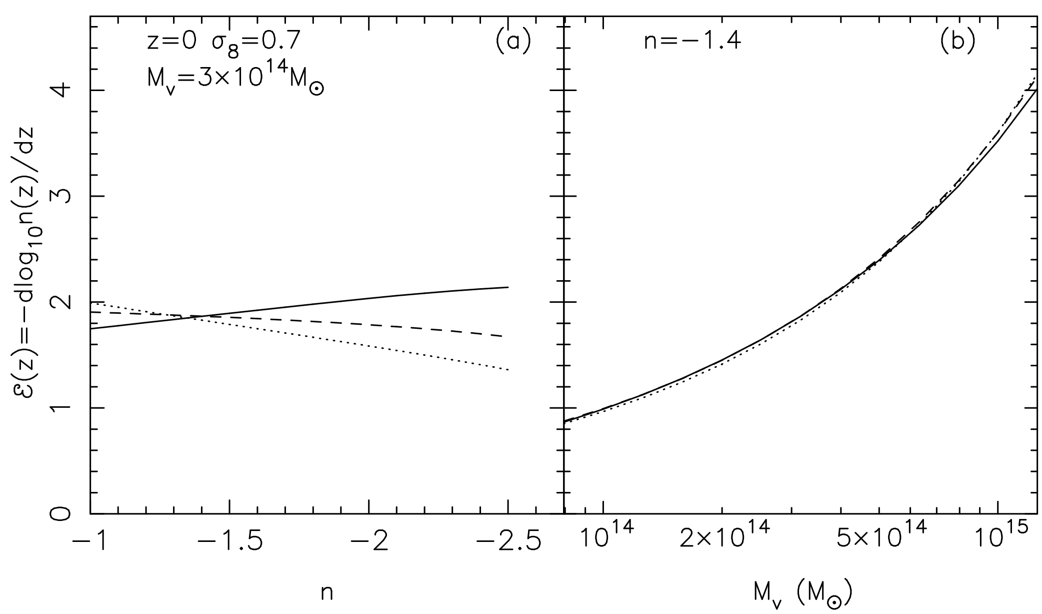

Figure 2. Dependence of the evolution rate $\mathcal{E}(z)$ on the slope of the power spectrum n and on the cluster mass threshold M. Solid, dashed and dotted lines correspond to $\Omega = 1$, 0.5 and 0.3 flat models, respectively.

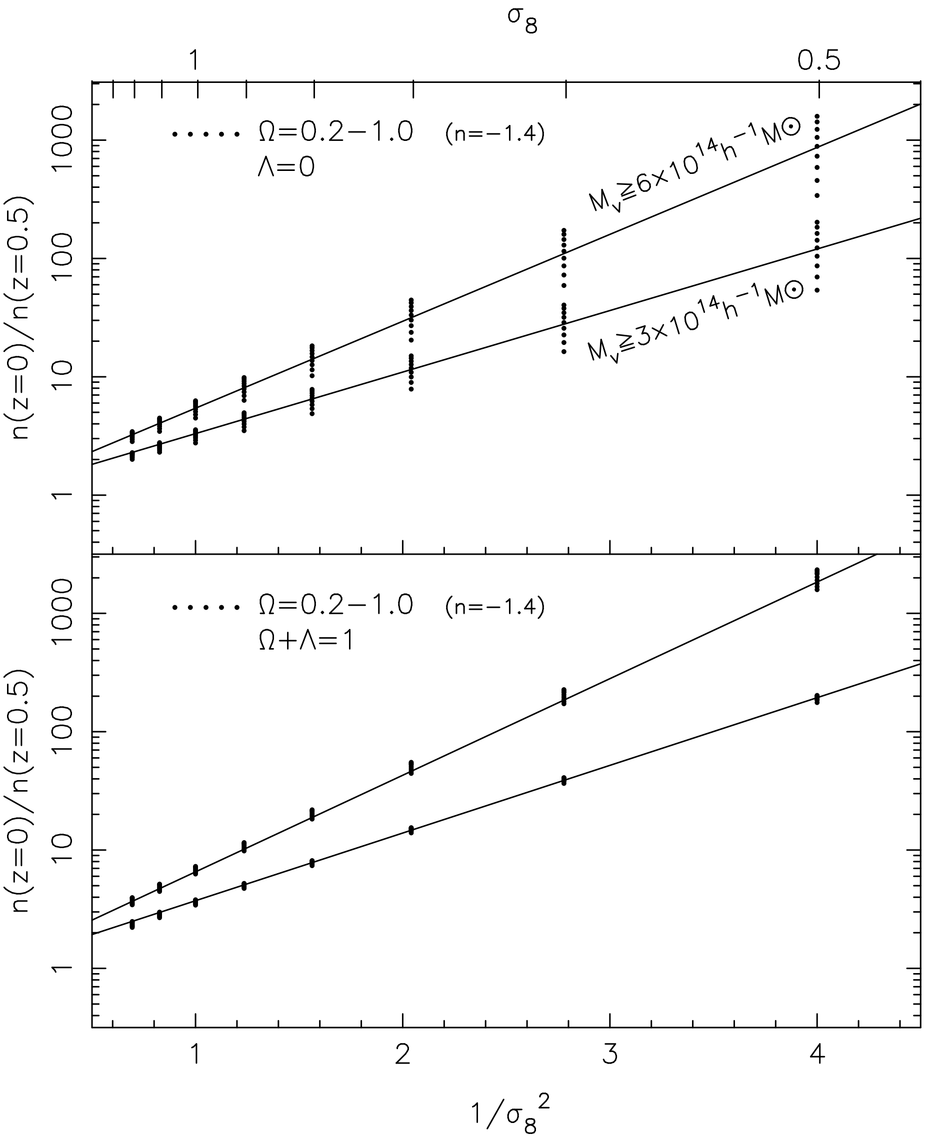

Figure 3. Cluster abundance ratio $n(z=0)/n(z=0.5)$ versus σ_8 for clusters with virial mass $M_v \geq 3 \times 10^{14}\mathrm{h}^{-1}\mathrm{M}_\odot$ (Richness $\gtrsim 1$), and $M_v \geq 6 \times 10^{14}\mathrm{h}^{-1}\mathrm{M}_\odot$ (Richness $\gtrsim 2$), from the Press-Schechter relation eq.(1) (represented by dots for all values of Ω from 0.2 to 1 (bottom to top) for each σ_8). The lines are the best-fit empirical relations (eq.9, with $\Omega \simeq 0.5$).

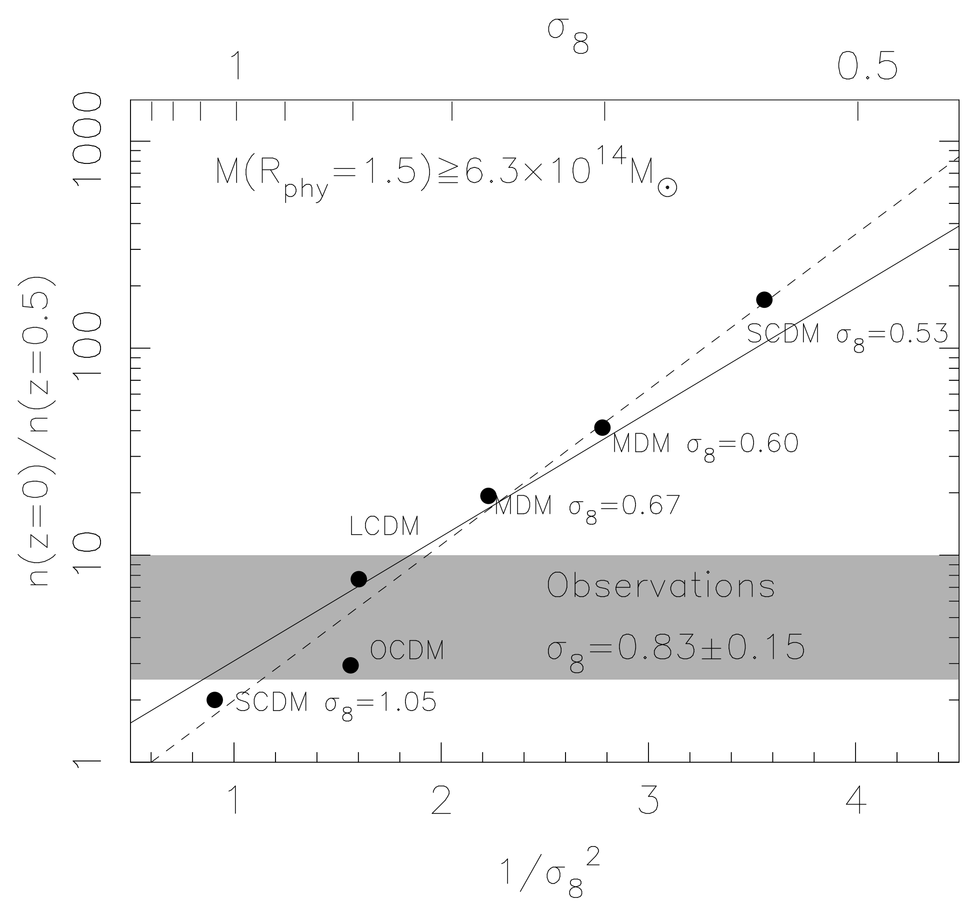

Figure 4. Cluster abundance ratio $n(z = 0)/n(z = 0.5)$ versus σ_8 from N-body simulations (dots), and Press-Schechter approximation (solid line), for $M_{1.5} \geq 6.3 \times 10^{14} \mathrm{h}^{-1} \mathrm{M}_{\odot}$ clusters. The observed abundance ratio (§3) is shown by the shaded region. (The solid line is the direct P-S relation (eq.1) for the given $M_{1.5}$ mass threshold, for $\Lambda = 0$ and mean of all Ω's; the dashed line represents the best-fit relation from the simulations.)